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Sources of Error Model and Progress Metrics for Acoustic/Infrasonic Analysis: Location Estimation

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Abstract—How well can we locate events using infrasound? This question has obvious implications for the use of infrasound within the context of nuclear explosion monitoring, and can be used to inform decision makers on the capability and limitations of infrasound as a sensing modality. This paper attempts to answer this question in the context of regional networks by quantifying current capability and estimating future capability using an example regional network in Utah. This example is contrasted with a sparse network over a large geographical region (representative of the IMS network). As a metric, we utilize the location precision, a measure of the total geographic area in which an event may occur at a 95 % confidence level. Our results highlight the relative importance of backazimuth and arrival time constraints under different scenarios (dense vs. sparse networks), and quantify the precision capability of the Utah network under different scenarios. The final section of this paper outlines the research and development required to achieve the estimated future location precision capability.

Key words: Infrasound, event location, nuclear explosion monitoring.

1. Introduction

Infrasound has been used for decades as a sensor modality to monitor atmospheric nuclear tests. Within the last ten years, the deployment of the international monitoring system (IMS) infrasound network under the comprehensive nuclear-test-ban treaty (CTBT), in addition to successes from prototype regional networks, is highlighting the additional value of infrasound as a supplemental tool for monitoring underground tests. With the drive towards monitoring low yield tests at regional distances, infrasound has the potential to play a key role in identifying mining blasts and other manmade events that become prevalent at these low seismic magnitudes. In addition, infrasound can provide constraints on source depth, which can be critical for reliably estimating the yield from seismic data and for source identification purposes.

Within the context of treaty monitoring, it is important to communicate to decision makers the capability of different technologies. Recent papers (LE PICHON *et al.* 2009; GREEN and BOWERS, 2010) have quantified the detection capability of the IMS infrasound network in terms of yield thresholds. However, there has not yet been any attempt to quantify what is achievable with regional networks of infrasound arrays, or to formally quantify localization uncertainty. With the recent deployment of a prototype infrasound network in Utah, it is possible to provide a preliminary assessment of the capability. While the capability will depend on the network configuration (e.g., number and density of stations), which will differ from network to network, there is value in providing an assessment of the Utah network for decision makers as an example scenario. The results will both quantify the current state-of-art capabilities in localization, as well as project what might be achievable in the future by leveraging research advances.

The purpose of this paper is to outline a formal approach for assessing network location precision for decision makers. Previous studies at IMS scales have assessed location accuracy using some measure of the azimuthal gap. However, in treaty monitoring applications the question of more relevance is how large a geographic area a given test is constrained within. For onsite inspection activities, areas of up to 1,000 km^2 are permitted. We discuss a methodology for estimating location precision for a regional infrasound

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network, then illustrate both existing capability and an estimate of future capability using the Utah infrasound network as an example. Finally, we outline the research needed to move from the first to the second scenario.

This paper uses the Bayesian infrasound source location (BISL) (Modrak *et al.*, 2010) to formally assess location precision. The advantage of BISL as compared with other techniques for this purpose is that both measurement error and model error are formally accounted for, and prior constraints can be readily folded into the solution. As a forward method, BISL contrasts with the standard inverse approach that is used for infrasound location (Brown *et al.*, 2002a; Ceranna *et al.*, 2009). Such inverse techniques are based on Geiger's method (Geiger, 1912). For using azimuthal information in addition to arrival time constraints, Bratt and Bache (1988) outline a formulism that has been used by researchers in the infrasound community (Ceranna *et al.*, 2009). Brown *et al.* (2002b) and Evers *et al.* (2007) implemented methods where bearing intersections were weighted by the sine of the angle of intersection. An innovative approach for event location that uses a space–time approach has also been developed (Szuberla and Arnoult, 2011; Szuberla *et al.*, 2009) but is not considered in this study because it is applicable for near-source recordings and not to events recorded at regional and global distances.

2. Methodology

2.1. The Bayesian Infrasonic Source Locator: A Recap

Modrak *et al.* (2010) introduced the Bayesian infrasonic source location (BISL) method for localizing infrasound events using a regional network of infrasound arrays. BISL uses both arrival time and back azimuth constraints to provide credibility bounds (analogous to confidence bounds in frequency statistics) on event location. This probabilistic approach accounts for uncertainties in both measurement and model error relevant to the infrasound location problem. We can represent the uncertainties associated with each parameter by:

$$\sigma(\theta)_{\text{total}} = \sigma(\theta)_{\text{measured}} + \sigma(\theta)_{\text{model}}$$
$$\sigma(\phi)_{\text{total}} = \sigma(\phi)_{\text{measured}} + \sigma(\phi)_{\text{model}}$$

where, θ represents back azimuth and ϕ represents arrival time.

As a crude estimate of the total uncertainty in back azimuth, $\sigma(\theta)_{\text{total}}$, we can make no correction for back azimuth deviation due to wind bias, and simply resort to empirical observations of back azimuth deviations to define the uncertainty. As a first order enhancement we can expect that propagation modeling would allow us to correct the azimuths for wind bias and reduce the corresponding model uncertainty. For arrival times, we may assume that phase is unknown and different at each array; therefore $\sigma(\phi)_{\text{model}}$ should be sufficiently large to capture the range of possible arrival times at the arrays. Similarly, as a first-order enhancement, we can utilize a distance-dependent probability density function on group velocity, with additional levels of sophistication that may include azimuthal and temporal dependence. Such an approach is currently being explored by Marcillo *et al.* (2012). These choices and level of sophistication affects the subsequent model error.

These uncertainties are implemented in the calculation of the likelihood, assuming Gaussian distributed errors, over all possible locations (latitude, longitude), origin times (t_0), and group velocities (v). Following Modrak *et al.* (2010), the likelihood is calculated from:

$$P(d|m) = \prod_{i=1}^{n} \Theta_i(\theta_i|m)\Phi_i(t_i|m)$$

where,

$$\Theta_i(\theta_i|m) = \frac{1}{\sqrt{2\pi\sigma_\theta^2}} \exp\left[-\frac{1}{2}\left(\frac{\gamma_i}{\sigma_\theta}\right)^2\right]$$
$$\Phi_i(t_i|m) = \frac{1}{\sqrt{2\pi\sigma_\phi^2}} \exp\left[-\frac{1}{2}\left(\frac{\varepsilon_i}{\sigma_\phi}\right)^2\right],$$

represent the individual likelihood components for the backazimuths and arrival times, respectively. With $d_i = d_i(x_0, y_0, x_i, y_i)$ as the distance from each hypothetical source to the i'th array, and for Cartesian coordinates, the residual terms are:

$$\gamma_i \equiv \theta_i - \arctan\left(\frac{y_i - y_0}{x_i - x_0}\right)$$
$$\varepsilon_i \equiv t_i - \left(t_0 + \frac{d_i}{v}\right).$$

Location solutions are computed from the multivariate posterior probability distribution of location parameters. A uniform prior on the group velocity is used in our initial development of BISL (that is, the group velocity can vary over some range, typically chosen to represent the possible range of infrasonic group velocities at a particular distance scale). The limitation of the initial development, as outlined in Modrak *et al.* (2010), is the assumption that the group velocity is the same for each array in the model. In practice, different phases will typically be observed in different directions due to wind, which introduces anisotropy not accounted for in our preliminary model. At present, we account for the model anisotropy through the standard deviation in arrival time, which is implemented in the likelihood equations. As long as this standard deviation is set sufficiently large to capture the possible range of arrival times for different phases, the model error is adequately accounted for. Future development of BISL will include construction of source-path specific priors for group velocity including priors based on atmospheric predictions of group velocity with associated errors. These improvements should better account for model error through explicitly accounting for anisotropy and by incorporating different priors to different stations (this is discussed in the last section of this paper).

2.2. A Location Metric

Event location capability should ideally be estimated by the precision of a location estimate (this is the area of uncertainty of the location ellipse or polygon). If both measurement and model errors are adequately estimated, the location ellipse or polygon should enclose the actual event location on most occasions (the credibility or confidence level at which the ellipse or polygon is estimated should define the percentage of times the solution will not enclose the location). Previous studies of infrasound location capability, which have focused on the IMS network, have represented location capability by an estimate of the accuracy based on the azimuthal separation between arrays for a given source location (e.g., Le Pichon *et al.*, 2009; Green and Bowers, 2010). Location accuracy (expressed in km) can be differentiated from precision (expressed in km^2). Within the context of the CTBT, where onsite inspection activities can occur inside an area of 1,000 km^2, precision is a more useful measure (assuming, of course, that the model uncertainties are appropriately defined).

Using BISL as our location technique, precision can be calculated in a straightforward manner by calculating the area enclosed by a location polygon at a specified credibility. These polygons are derived from the posterior distribution of location parameters, where the posterior integrating constant is derived with numerical integration, with the initial model formulation. In this study, we use a credibility of 0.95 for calculating the polygons, because this corresponds to a typical level at which confidence regions are calculated. Three different scenarios are considered: (1) a best case scenario where each event is detected at every array, (2) a 'typical' northern hemisphere winter scenario where the stratospheric wind jet blows towards the east, and (3) a 'typical' northern hemisphere summer scenario where the stratospheric wind jet blows towards the west. For scenarios 2 and 3 we assume that no arrivals are observed at distances greater than 200 km in the counter-wind direction (i.e., no stratospheric arrivals are observed in the counter-wind direction). However, within 220 km, tropospheric arrivals are observed 50 % of the time, regardless of direction. Although these assumptions are simplistic, the resultant maps provide a more realistic assessment of the location capability than the simulation shown in Fig. 1, and also provide a sense of the effect of seasonal variations.

3. The Utah Infrasound Network

The Utah infrasound network was deployed in two phases. In 2006, three infrasound arrays were installed around Great Salt Lake to record infrasound signals from rocket motor detonations at the

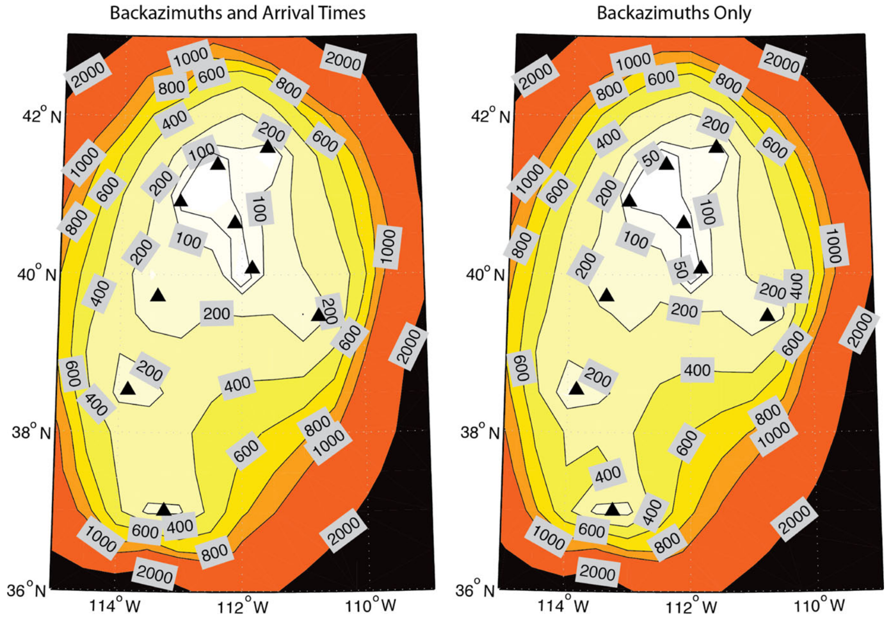

Figure 1
Simulation results showing location uncertainty (in km^2) as a function of location for the Utah infrasound network. Based on the arguments outlined in the paper, these values are representative of current localization capability

Utah Test and Training Range (UTTR) (Stump *et al.*, 2007). In 2008–2009, the network was extended to include six new arrays in order to study infrasound from small earthquakes in the intermountain seismic belt. The full 9-array network is an excellent test bed for research into the capability of high-density regional infrasound networks. The locations of the arrays in the network are provided in Table 1 and shown as black triangles in the figures.

Table 1

Locations of arrays in the Utah infrasound network

Array name	Latitude	Longitude
BGU	40.9204	−113.0309
BRP	39.4727	−110.7409
EPU	41.3901	−112.4099
FSU	39.7196	−113.3900
HWU	41.6071	−111.5642
LCM	37.0109	−113.2444
NOQ	40.6526	−112.1186
PSU	38.5332	−113.8555
WMU	40.0795	−111.8310

4. Results

4.1. The Utah Infrasound Network

The location precision estimates have been computed for two scenarios. The first scenario represents an estimate of current capability. The second scenario attempts to assess what might be possible in the future given further research (we discuss the research that is required to fulfill these goals as outlined in the following section). Each scenario is tested and applied on the Utah infrasound network (representative of a spatially-dense network).

Under the first scenario (current operational capability) the following assumptions are made. First, for back azimuth we assume that the existing sum of measurement and model error, $\sigma(\theta)_{\mathrm{total}}$, at any given array can be represented by a standard deviation of 3°. This value is consistent with recent empirical observations of tropospheric and stratospheric signals from ground-truth explosions at the Hawthorne Army Ammunition Depot in Nevada (Negraru, Pers. Comm.) in addition to azimuth deviations observed from measurements of earthquakes in the western

USA (Mutschlecner and Whitaker, 2005). Second, we assume that the identification of phase is non-unique, and therefore that group velocity could vary from 0.28–0.34 km/s (neglecting thermospheric returns). For the size of the Utah network this could result in a spread as large as $\sim$180 s (this value is calculated from the maximum distance between all pairs of arrays and the subsequent difference in travel time for the mean group velocity of 0.31 km/s and the slowest group velocity of 0.28 km/s). Setting $\sigma(\theta)_{\text{total}}$ equal to 100 s, it is clear that, for this network configuration, back azimuth dominates the location capability (Fig. 1). On this basis, the most significant improvements to current capability can be made by reducing the back azimuth model error. The measurement error for back azimuth, typically quoted as $\sim$0.5° but dependent on the array configuration and processing parameters, is small in comparison to model error (Szuberla and Olson, 2004). Of course, this conclusion is based on the Utah network, which is a relatively dense infrasound network. For sparse networks, the importance of arrival time on the solution should be greater.

As shown in Fig. 1, location precisions of 50 km^2 are possible in the densest portion of the network. The whole network region is enclosed by the 400 km^2 contour, indicating that any event within this region will have a location precision better than this value. As one moves away from the region spanned by the network, the location precision degrades as expected. Most notably, the location precision does not improve by adding arrival time constraints with the large model error used. For comparison, under the second scenario (future capability), we assume that a realistic attainable improvement in the total back azimuth uncertainty is $\sigma(\theta)_{\text{total}} = 1.5°$ and that a realistic attainable improvement in arrival time estimation is 20 s (Hedlin *et al.*, 2011). We expect that these improvements will be primarily driven by the reduction of model error. The resultant simulations, shown in Fig. 2, can be compared with the simulations shown in Fig. 1. The improvement in localization precision is dramatic, with the network now enclosed by a precision of 50 km^2. Additionally, it is clear that although the dominant effect is back azimuth, the inclusion of arrival times does improve the location capability at these smaller model errors.

The simulations of typical summer and winter scenarios, using estimates of present capability, are shown in Fig. 3. These plots represent averages of ten realizations for each season (as discussed above, each realization has a random distribution of observations, with 50 % of arrays detecting signals inside 220 km). These results provide a more realistic estimate of location precision, and how it differs at different times of the year. Location precisions of <400 km^2 are possible close to individual arrays, with a more typical location precision of $\sim$800 km^2 surrounding most of the network. During the summertime, since the stratospheric winds blow towards the west in the northern hemisphere mid-latitudes, the network does not detect events at distances greater than 220 km to the west. This scenario is reversed during winter when the stratospheric winds at these latitudes blow towards the east.

4.2. The IMS Network: A Test for the European Region

For comparison to small regional scales, consider a large region with spatially limited stations. For this case, arrival times are much more important, as is illustrated for the two example events in Fig. 4. The top panel in Fig. 4 represents current *operational capability* ($\sigma(\phi)_{\text{total}} = 1{,}000$ s to account for any phase at any array, and $\sigma(\theta)_{\text{total}} = 3^{\circ}$) while the bottom panel represents future capability ($\sigma(\phi)_{\text{total}} = 100$ s to account for any phase at any array, and $\sigma(\theta)_{\text{total}} = 1.5°$). We note that by operational capability we refer to automatic or real-time methods. The improvements suggested by the future scenario maps require enhancements to the BISL algorithm, in addition to enhancements in propagation models, as outlined below.

5. Research and Development Needed for Location Improvements

This paper argues that, for dense deployments like the Utah network, key enhancements to location capability will come through propagation modeling that can reliably predict the azimuthal deviation caused by the propagation of infrasound from source

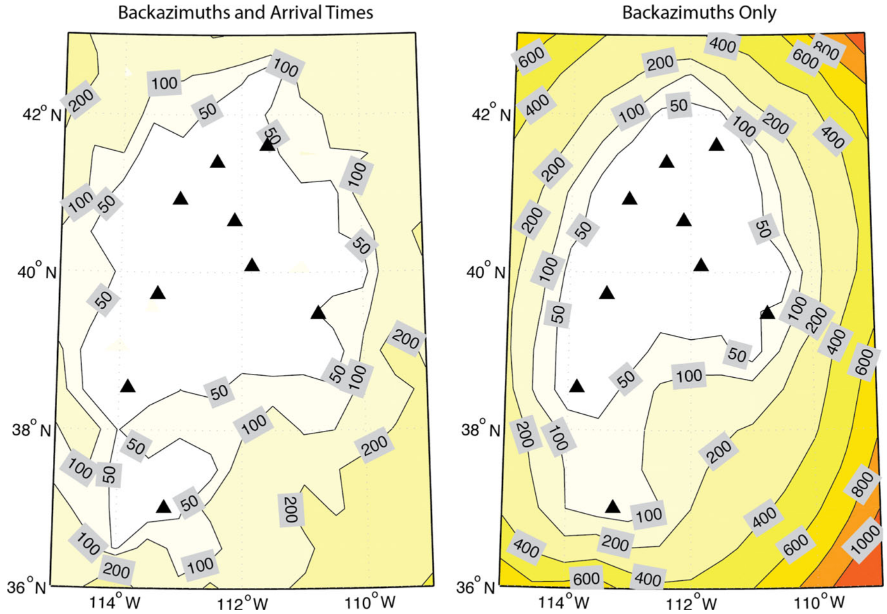

Figure 2
Simulation results showing potential localization uncertainty (in km^2). Based on the arguments outlined in the paper, these values are representative of potential future localization capability

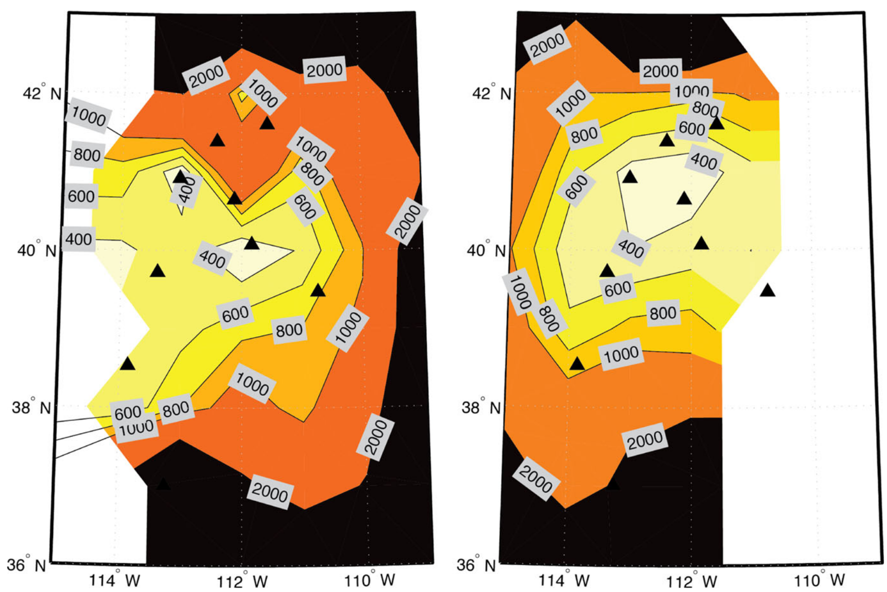

Figure 3
Summer (*left*) and winter (*right*) maps using $\sigma(\theta)_{\mathrm{total}} = 3°$ and $\sigma(\phi)_{\mathrm{total}} = 100$ s. *Colors* represent the location precision in km^2. Areas *shaded white* denote regions where $\leq$1 station would detect an event, and therefore localization using BISL is not possible

to receiver. The prior on azimuth is constructed by embedding physical azimuthal predictions into the appropriate error model. For sparse network configurations, however, enhancements to the localization algorithm are needed that can incorporate physics-based priors and account for propagation anisotropy. The steps required to meet these enhancements can be broken down into three categories, which are

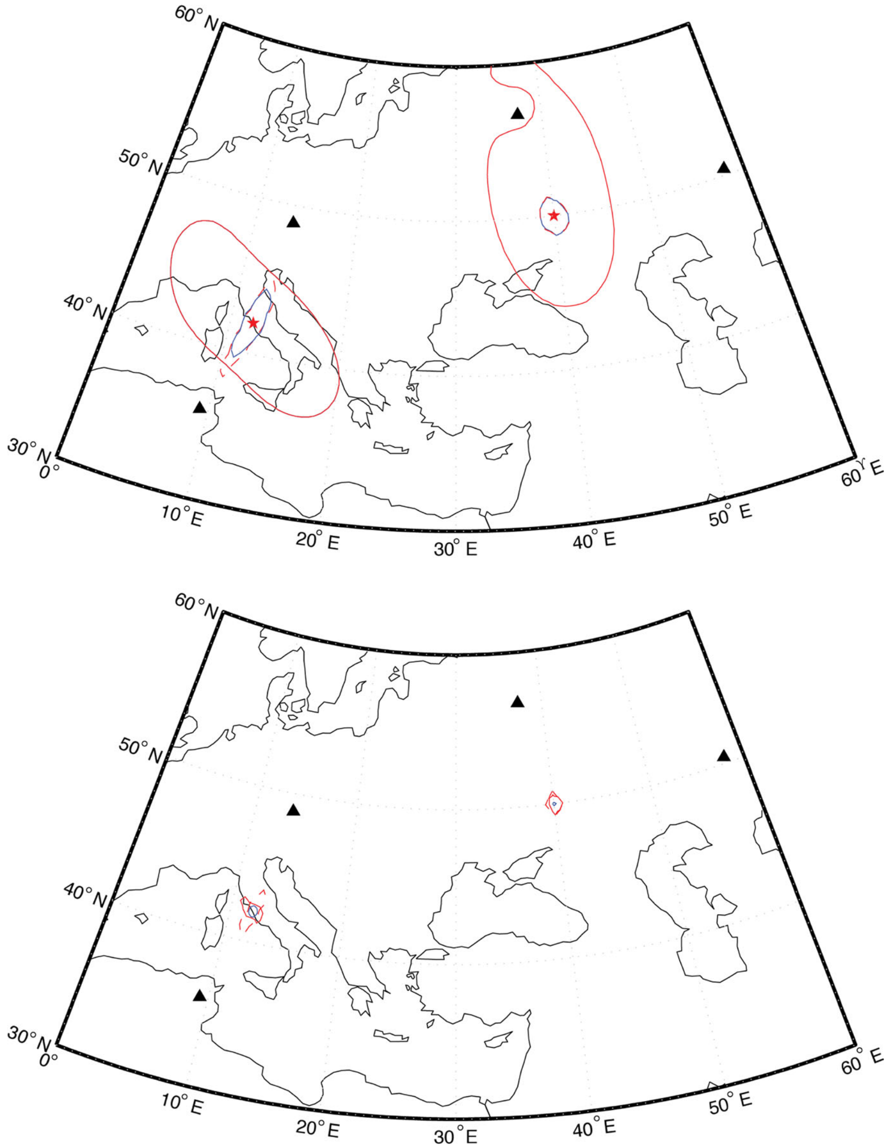

Figure 4
Location simulations for two hypothetical events in Europe with four IMS arrays

discussed separately below. A final major enhancement can be made through combining seismic and infrasonic data for the combined geolocation of seismo-acoustic events; this is discussed as a fourth category.

5.1. Improvements to the Mathematics of the Location Framework

The current implementation of BISL assumes the same group velocity to each array in the network. Although the Bayesian prior accounts for the unknown group velocity, it does not adequately account for different phase arrivals at different arrays; this scenario is currently accounted for by increasing the standard deviation in arrival times to include a model error component. Model anisotropy may be a second-order effect at global distances where stratospheric returns are the predominant arrival, and such arrivals are typically observed along similar azimuths in the direction of stratospheric wind. However, at regional distances, where tropospheric returns are common, model anisotropy is more pronounced. Further research and development of BISL is required to enable the inclusion of more sophisticated model predictions, incorporating state-of-the-art propagation models and atmospheric specifications, in the form of Bayesian priors (Marcillo *et al.*, 2012). The Utah network simulations suggest that back azimuths are the dominant effect on the location precision; however, for global scale monitoring using the IMS network, where perhaps only two arrays will detect an event, the inclusion of arrival times will be essential. In this scenario, refinements to the existing BISL framework are expected to lead to dramatic improvements in location precision. The key refinement needed is the inclusion of an array-specific group velocity; in other words ε_i in the likelihood equation would equal:

$$\varepsilon_i = t_i - \left(t_0 + \frac{d_i}{v_i}\right).$$

Included in this framework advance is the development of BISL-specific algorithms to derive the multivariate posterior distribution of location parameters, specifically a Gibb's sampling algorithm (Casella and George, 1992). This development will enable the use of BISL in a near-real-time operational setting.

Another case where travel time information has a large impact on performance is sparse or poorly distributed networks, regardless of their spatial extent. Poor azimuth-only localizations occur when the arrays are grouped along similar azimuthal directions with respect to the source. The resulting localization uncertainty ellipse is very narrow along its major axis, elongated along the direction to the arrays. The quantification of this effect is achieved using geometric dilution of precision (GDOP) (Yarlagadda *et al.*, 2000; Levanon, 2000). GDOP metrics are derived by quantifying the sensitivity of source localization to array measurements, and good GDOP is supported when the arrays span a variety of azimuthal directions.

In the case of poor GDOP, travel time information can supplement the azimuthal data and result in significantly improved localization precision. This improvement occurs because the uncertainly ellipses for travel-time only localizations are elongated orthogonal to that for azimuth. Therefore, when the two measurements are combined, the localization is more tightly constrained, with an uncertainty ellipse that is more circular in nature. [As an example, see Norris and Gibson (2002)].

5.2. *Improvements to Propagation Models, Atmospheric Specifications*

For predicting arrival times and back azimuth deviations at regional distances (<2,000 km), 3D ray tracing is state-of-the-art. To first order, existing implementations do a reasonable job at predicting arrivals beyond the zone of silence with state-of-the-art 4D atmospheric specifications (e.g., the Ground-to-Space model, Drob *et al.*, (2003)). There are however, improvements to the propagation models that would increase the fidelity of the predictions by accounting for additional physics.

As ray tracing is a geometrical approximation, it does not account for frequency dependent effects. The model is applicable as long as the acoustic wavelength is small in comparison with the physical scales of the medium (Kinsler *et al.*, 1982; Jensen *et al.*, 1994). As this assumption breaks down, the resulting effect for infrasound propagation is that we begin to observe geometric dispersion. Each frequency component takes a slightly different propagation path and there is a spreading of the wave packet as observed at the array. Time-domain parabolic equation models (TDPE) (Norris *et al.*, 2007) can be applied along a given azimuth to predict this effect. Higher fidelity travel time predictions could then be made by comparing the observed and predicted waveforms, where the same phase arrival (peak picking) algorithm would be applied.

Although the TDPE approach can be used for travel times, it does not account for deviations along a given azimuth, which the 3-D ray model quantifies with the azimuthal deviation metric. Research could be pursued to account for these effects using full-wave models. The approach could either be based on exploring 3-D implementations, or in developing hybrid N by 2D solutions.

Another area of research is in the prediction of the propagation paths (phases). The well-established paths of interest are capped at the top of the troposphere and stratosphere. Quantifying their stability, strength, and evolution can be captured with the high-resolution atmospheric specification proposed below, coupled with Monte Carlo simulations. An additional propagation path may also be relevant: infrasonic energy gets trapped in the stratopause and "rides" this height for an extended distance before ultimately escaping and refracting back down to the ground. The trapping occurs as the energy bounces between layers of inhomogeneities driven by gravity waves. This path is significant because the group velocities are much larger than those for a stratospheric path. More research is needed in further quantifying the condition needed and frequency to which this additional propagation path occurs.

For the atmospheric specification, at distances inside 200 km, the addition of fine-scale structure is typically required in order to predict observed arrivals. The key research required, associated with improvements to propagation models and atmospheric specifications *for location purposes*, is related to the incorporation of this fine-scale structure and its effect on the prediction of travel times and back azimuth deviations. Research is needed on the implementation of such methods for the prediction of Bayesian prior PDF's on azimuthal deviations and arrival times.

Recent work has illustrated the value in incorporating spectral models for gravity waves in propagation simulations (Kulichkov, 2004; Gibson *et al.*, 2009). With the use of such stochastic models, Monte Carlo methods are needed to adequately sample the solution space. Due to the dynamic nature of the atmosphere, the solution space is spanned both spatial and temporally. Predictions that capture the relevant scales are on the order of 0.1–1 degree in space and hourly in time. Obviously this modeling fidelity must be balanced with the resulting payoff and computation loads they introduce. The modeling turn-around time for analysis and operational groups must also be considered. With these issues in mind, research in optimization of the front-to-end modeling chain would also be beneficial.

In all cases, these more sophisticated group velocity and back azimuth models are predictions of stochastic atmospheric processes and therefore have error. Embedding these predictions into the appropriate error model provides the priors for the advanced framework.

5.3. *Validation of Propagation Models, Atmospheric Specifications*

Perhaps the most neglected area of research to date has been the adequate validation of propagation models using large ground-truth datasets (comprising 1,000's of arrivals). In part, this has been a result of the limited infrasound data openly available to researchers. Recent work with USArray data has illustrated how spatially-aliased most infrasound networks are, and how USArray is enabling such validation for the first time (Hedlin *et al.*, 2010). Unfortunately, USArray comprises single acoustic elements and therefore cannot be used to validate model predictions of back azimuth deviations. To address this limitation, research deployments like the Utah network are essential for gathering sufficient ground-truth data with which to properly test models. Such deployments are in their infancy, and little research has been done to date on this aspect. However, based on the findings presented in this paper, such work is critical to validate the model implementations, which will ultimately improve the localization capability that can be obtained using regional infrasound networks.

5.4. *Seismo-acoustic Location*

The complementary nature of infrasound and seismic data argues for combining both datasets for simultaneous localization of seismo-acoustic events. Seismic data provide better traveltime constraints whereas infrasound back azimuths are superior owing to the relatively low lateral heterogeneity in the atmosphere. Che *et al.* (2009) demonstrate the advantage of combining both technologies by demonstrating how a combined seismo-acoustic location is more accurate, compared to both seismic and infrasound locations, for a ground-truth mining explosion in Korea. Pinsky *et al.* (2012) report on highly accurate seismo-acoustic locations for a series of explosions in Israel. These papers show promise but there has been little research on how to best combine seismic and infrasound data for localization.

Towards this goal, we plan to explore an extension of the BISL framework discussed in this paper through the inclusion of seismic traveltime constraints.

6. Conclusions and Future Research

We have discussed metrics for regional infrasound monitoring, and how these might be improved with improvements to existing atmospheric specifications and propagation models. Similar metrics are needed to guide the development of atmospheric specifications and infrasound propagation models (e.g., how well one can predict the arrival times and amplitudes of signals), but these are not the focus of this paper. Ultimately, for the CTBT monitoring problem, the key parameters are detection, location, and yield estimation. The authors are currently developing acoustic/infrasonic yield estimation methods that require good source location for path corrections. By providing metrics to quantify the capability of infrasound technology in estimating these parameters, the community can better provide decision makers with the tools they need to understand both existing and potential capability.

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References

Bratt, S.R. and Bache, T.C., 1988. *Locating Events with a Sparse Network of Regional Arrays*, Bull. Seism. Soc. Am., *78*, 780–798.

Brown, D.J., Katz, C., Le Bras, R., Flanagan, M.P., Wang, J. and Gault, A.K., 2002a. *Infrasonic Signal Detection and Source Location at the Prototype International Data Center*, Pure appl. geophys., *159*, 1081–1125.

Brown, P., Whitaker, R.W., Revelle, D. and Tagliaferri, E., 2002b. *Multi-station infrasonic observations of two large bolides: signal interpretation and implications for monitoring of atmospheric explosions,* Geophys. Res. Lett., *29*. doi:10.1029/2001GL013778.

Casella, G. and George, E., 1992. *Explaining the Gibbs sampler,* The American Statistician, *46*, 167–174.

Ceranna, L., Le Pichon, A., Green, D.N. and Mialle, P., 2009. *The Buncefield explosion: a benchmark for infrasound analysis across Central Europe*, Geophys. J. Int., *177*, 491–508.

Che, I.-Y., Shin, J.S. and Kang, I.B., 2009. *Seismo-acoustic location method for small-magnitude surface explosions,* Earth Planets Space, *61*, e1-e4.

Drob, D.P., Picone, J.M. and Garces, M., 2003. *Global morphology of infrasound propagation*, J. Geophys. Res., *108*. doi:10.1029/2002JD003307.

Evers, L.G., Ceranna, L., Haak, H.W., Le Pichon, A. and Whitaker, R.W., 2007. *A Seismoacoustic Analysis of the Gas-Pipeline Explosion near Ghislenghien in Belgium*, Bull. Seism. Soc. Am., *97*, 417–425. doi:10.1785/0120060061.

Geiger, L., 1912. *Probability method for the determination of earthquake epicenters from the arrival time only*, Bull. St. Louis. Univ., *8*, 60–71.

Gibson, R., Drob, D.P. and Broutman, D., 2009. *Advancement of Techniques for Modeling the Effects of Fine-scale Atmospheric Inhomogeneities on Infrasound Propagation.* in 2009 Monitoring Research Review, Tucson.

Green, D.N. and Bowers, D., 2010. *Estimating the detection capability of the International Monitoring System infrasound network,* J. Geophys. Res., *115*. doi:10.1029/2010JD014017.

Hedlin, M.A.H., De Groot-Hedlin, C.D. and Drob, D.P., 2011. *A Study of Infrasound Propagation Using Dense Seismic Networks American Geophysical Union Fall Meeting, Abstract A31A-0037*, Bull. Seism. Soc. Am., Submitted.

Hedlin, M.A.H., Drob, D.P., Walker, K.T. and De Groot-Hedlin, C.D., 2010. *A study of acoustic propagation from a large bolide using a dense seismic network*, J. Geophys. Res., *115*. doi:10.1029/2010JB007669.

Jensen, F., Kuperman, W., Porter, M., Schmidt, H., 1994. *Computational Ocean Acoustics, AIP Press, Woodbury, NY, Sec. 3.3.1.*

Kinsler, L, Frey, A., Coppens, A. and Sanders, J. 1982. *Fundamentals of Acoustics, 3rd. Ed., Wiley, NY, Sec. 5.13.*

Kulichkov, S.N., 2004. *Long-range propagation and scattering of low-frequency sound pulses in the middle atmosphere*, Meteor. and Atmos. Phys., *85*, 47–60. doi:10.1007/s00703-003-0033-z.

Le Pichon, A., Vergoz, J., Blanc, E., Guilbert, J., Ceranna, L., Evers, L.G. and Brachet, N., 2009. *Assessing the performance of the International Monitoring System's infrasound network: Geographical coverage and temporal variabilities*, J. Geophys. Res., *114*. doi:10.1029/2008JD010907.

Levanon, N., 2000. *Lowest GPS in 2-D scenarios*, IEE Proc.-Radar, Sonar Navig., *147* (3), 149–155.

Marcillo, O., Arrowsmith, S.J., Whitaker, R.W. and Anderson, D.N., 2012. *Enhancements to the Bayesian Infrasound Source Location Method.* in 2012 Monitoring Research Review, Albuquerque, NM.

Modrak, R.T., Arrowsmith, S.J. and Anderson, D.N., 2010. *A Bayesian framework for infrasound location*, Geophys. J. Int., *181*, 399–405. doi:10.1111/j.1365-246X.2010.04499.x.

Mutschlecner, J.P. and Whitaker, R.W., 2005. *Infrasound from earthquakes*, J. Geophys. Res., *110*. doi:10.1029/2004JD005067.

Norris, D., Bhattacharyya, J. and Whitaker, R.W., 2007. *Development of Advanced Propagation Models and Application* *to the study of Impulsive Infrasonic Events*. in 29th Monitoring Research Review, edn 2007. Denver, CO.

NORRIS, D. and GIBSON, R., 2002. *InfraMAP Enhancements: Environmental/Propagation Variability and Localization Accuracy of Infrasonic Networks*. in 24th Seismic Research Review, pp. 809–813, Ponte Vedra Beach, Florida.

PINSKY, V., GITTERMAN, Y., BEN HORIN, Y. and ARROWSMITH, S.J., 2012. *Seismo-acoustic analysis for series of ammunition demolition explosions at Sayarim, Israel*. in European Geophysical Union General Assembly, Vienna, Austria.

STUMP, B., BURLACU, R., HAYWARD, C., PANKOW, K.L., NAVA, S., BONNER, J., HOCH, S., WHITEMAN, D., FISHER, A., KIM, T.S., KUBACKI, R., LEIDIG, M., BRITTON, J., DROBECK, D., O'NEILL, P., JENSEN, K., WHIPP, K., JOHANSON, G., ROBERSON, P., READ, R., BROGAN, R. and MASTERS, S., 2007. *Seismic and Infrasound Energy Generation and Propagation at Local and Regional Distances: Phase I—Divine Strake Experiment*. Air Force Research Laboratory.

SZUBERLA, C.A.L. & ARNOULT, K., 2011. Locating explosions, volcanoes, and more with infrasound, *Physics Today, 64*, 74–75. doi:10.1063/1.3580503.

SZUBERLA, C.A.L. and OLSON, J.V., 2004. *Uncertainties associated with parameter estimation in atmospheric infrasound arrays*, J. Acoust. Soc. Am., *115*, 253–258.

SZUBERLA, C.A.L., OLSON, J.V. and ARNOULT, K., 2009. *Explosion localization via infrasound*, JASA Express Letters, *126*. doi: 10.1121/1.3216742.

YARLAGADDA, R., ALI, I., AL-DHAHIR, N. and HERSHEY, J., 2000. *GPS GDOP metric*, IEE Proc.−Radar, Sonar Navig., *147* (5), 259–264.

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