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Multivariate acoustic detection of small explosions using Fisher's combined probability test

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Abstract: A methodology for the combined acoustic detection and discrimination of explosions, which uses three discriminants, is developed for the purpose of identifying weak explosion signals embedded in complex background noise. By utilizing physical models for simple explosions that are formulated as statistical hypothesis tests, the detection/discrimination approach does not require a model for the background noise, which can be highly complex and variable in practice. Fisher's Combined Probability Test is used to combine the *p*-values from all multivariate discriminants. This framework is applied to acoustic data from a 400 g explosion conducted at Los Alamos National Laboratory.

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1. Introduction

The detection of acoustic signals from explosions is a problem that can be trivial in the case of large explosions recorded in the near field, but quickly becomes more challenging as either the explosion size decreases or the offset range increases. For small and/or distant explosions, we require sophisticated detectors that exploit the different physical characteristics of acoustic signals from explosions in order to identify such signals amongst background noise (which may be larger in amplitude and contain some of the same physical characteristics). This same framework can also be used to distinguish between different types of explosions, where a physical acoustic signature manifests such differences.

Many detectors use a null hypothesis that is based on a background noise model where the noise is white and stationary (e.g., Kay, 1998). In reality, acoustic noise characteristics are difficult to determine *a priori* and, since we are interested in detecting a certain type of signal, we define null hypotheses that are based on the expected physical characteristics of small explosions: short-duration, impulsive, broadband, coherent, all without making any assumptions on the background noise except that is exists. Each detector is based on the same null hypothesis of signal plus noise and a probability model is constructed for each test statistic. Fisher's Combined Probability Test (Fisher, 1958) is then used to combine detections based on these separate physical properties into a single detection value.

The main point of this paper is to demonstrate an approach to the multivariate detector problem using a common hypothesis test, not the individual detectors. In practice, orthogonal detectors should be chosen, each measuring a different characteristic of the signal to be detected.

2. Methodology

This study explores the statistical combination of three physical discriminants for identification of small explosions in acoustic data in a multivariate setting. The discriminants are individual detectors that are designed around the expected signal characteristics from a small explosion recorded locally on an acoustic array or channel, so that we use the terms discriminants and detectors interchangeably. The unifying framework used in detector system design revolves around a single hypothesis-testing approach developed by Anderson *et al.* (2007) for the Event Classification Matrix currently used for teleseismic and regional discrimination of earthquakes and nuclear explosions. As stated by Anderson *et al.* (2007) in their article regarding multidimensional seismic discriminants, "For each discriminant a probability model is formulated under a general null hypothesis of H_0: *Explosion Characteristics*. The veracity of the hypothesized model is measured with a *p*-value calculation… A value near zero rejects H_0 and a moderate to large value indicates consistency with H_0. The hypothesis test formulation ensures that seismic phenomenology is tied to the interpretation of the *p*-value." Each discriminant is formulated in terms of a *p*-value, which provides a measure of the degree of membership of a given signal with the expected signal characteristics of small explosions.

So what is a *p*-value in the context of signal detection? A *p*-value can be thought of in many different ways, but in general it is the conditional probability that a test statistic, T_s, is as extreme as the one actually observed if the null hypothesis is true. In general, T_s for the signal exceeds that of the noise. By examining the mathematical details of each detector it is possible to find a closed-form probability distribution function (pdf) used to construct the *p*-values under the null hypothesis. The parameters of the pdf can be found using a set of calibration data and/or theoretical representations of the signal to be detected. In this paper, the signal will be that expected from a small explosion, so we can formulate the null hypothesis as

$$H_0 : \mathit{signal} + \mathit{noise} \tag{1}$$

(because there will always be noise contamination of the signal), where the signal has *explosion characteristics*. Assuming we can model the signal with a closed-form pdf, we define

$$p_d = P(H_0|T_s) = F(T_s) = \int_0^{T_s} f(T)\,\mathrm{d}T, \tag{2}$$

where p_d is the probability of detection under H_0, T_s is the observed test statistic, T is a random variable representing the test statistic, and $f(T_s)$ and $F(T_s)$ are the assumed pdf and cumulative distribution function (CDF) for T_s under H_0. Note that the noise pdf is never actually used but could be if the traditional binary hypothesis test involving likelihood ratios was desired. The reason we do not take this approach is because this requires a noise model, which may be complicated in practice as noise in our context is not pure Gaussian white noise. In fact, the noise includes background signals from a variety of other transient and continuous sources (e.g., vehicles, lightning, wind farms, etc.).

3. Individual discriminants (detectors)

Three detectors were utilized in this study: an array *F*, a short- to long-term average (STA/LTA), and a spectrogram detector. Each detector measures a different signal characteristic: signal coherence over an array over a short time span, a rapid increase in power, and a broadband short-duration signal.

The first discriminant is an array F-detector that measures the ratio of beam power on an array (where the beam is formed by time-aligning and stacking the waveforms from each array element over all directions of arrival such that the beam power is maximized) to the residual power (Blandford, 1974). By using a short time-window of 1 s, this discriminant identifies short-duration/high-frequency signals that may be consistent with explosions. The *F*-detector is formulated as

$$F = \left(\frac{J-1}{J}\right) \cdot \frac{\sum_{n=n_0}^{n_0+(N-1)} \left[\sum_{j=1}^{J} x_j(n+l_j)\right]^2}{\sum_{n=n_0}^{n_0+(N-1)} \left(\sum_{j=1}^{J} \left\{ x_j(n+l_j) - \left[\frac{1}{J} \cdot \sum_{m=1}^{J} x_m(n+l_m)\right]\right\}^2\right)}, \tag{3}$$

where J is the number of sensors, $x_j(n)$ is the waveform amplitude of the nth sample of the mean-free time-series from sensor j, l_j is the time-alignment lag obtained from beamforming, n_0 is the start sample index for the processing interval, and N is the number of samples in the processing window. Following Blandford (1974), under $H_0 : signal + noise$, F is distributed as a noncentral F-distribution such that, given $F = F_i$, we can calculate the p-value from

$$P\{F[N_1, N_2, \lambda(S/N)] \geq F_i\}, \tag{4}$$

where $N_1 = 2BT$, $N_2 = 2BT(N-1)$, and $\lambda = 2BT(S/N)^2$, where B is the bandwidth (Hz), T is the time-window (s), and S/N is the signal-to-noise ratio chosen for defining a detection. A p-value of <0.99 would indicate that H_0 : signal + noise for a given processing time window is rejected.

The STA/LTA and spectrogram detector are described in Taylor *et al.* (2010). The STA/LTA detector is simply a ratio of the mean square value of the signal in short time window (X_s^2) divided by a long time window (X_l^2). It is assumed that for a single channel, the STA/LTA follows a non-central F for H_0,

$$\frac{X_s^2}{X_l^2} \sim F(N_s, N_l, \lambda), \tag{5}$$

where $N_s = 2T_sB$ is the degrees of freedom for the numerator, $N_l = 2T_lB$ is the degrees of freedom for the denominator, T_s and T_l are the length of the short- and long-term windows, B is the bandwidth and $\lambda = N_s(S/N)^2$ is the non-centrality parameter where S/N is the signal-to-noise ratio. No assumptions regarding the noise power spectrum are made.

For the spectrogram detector of Taylor *et al.* (2010), small impulsive signals are manifest as vertical stripes on spectrograms (Fig. 1) and can be enhanced on gray-scale representations using vertical detection masks. Bitmap images are formed where pixels above a defined threshold are set to one. A short-duration, large bandwidth signal will have a large number of illuminated bits in the column corresponding to its arrival time. The marginal distribution of bit counts as a function of time is formed, n_i, by summing column wise over frequency. For each time window a hypothesis test, H_0: *signal* + *noise* is formed by defining a background density of ones, ρ_1, expected when a signal is present. The density represents the probability of success in a signal window assuming that n_i is a Bernoulli random variable. It is assumed that n_i follows the binomial distribution, allowing us to compute a probability of detection (represented as a p-value) for a given ρ_1.

To construct the null hypothesis of the form H_0: *signal* + *noise*, the p-value indicates the probability of detection computed from the binomial CDF as

$$p_d = F_B(n_i | n_f, \rho_1) = \sum_{i=0}^{n_i} \binom{n_f}{i} \rho_1^i (1-\rho_1)^{n_f - i}, \tag{6}$$

which is the probability of observing up to n_i bits in n_f independent trials.

By the probability integral theorem, the p-values for signal detection will, in general, be uniformly distributed between 0 and 1 under the null hypothesis of H_0: *signal* + *noise* (Rohatgi, 1976). The p-values can be treated as random variables drawn from a uniform distribution on the interval [0,1]. A data fusion method based upon Fisher's Combined Probability Test is then used by defining the test statistic

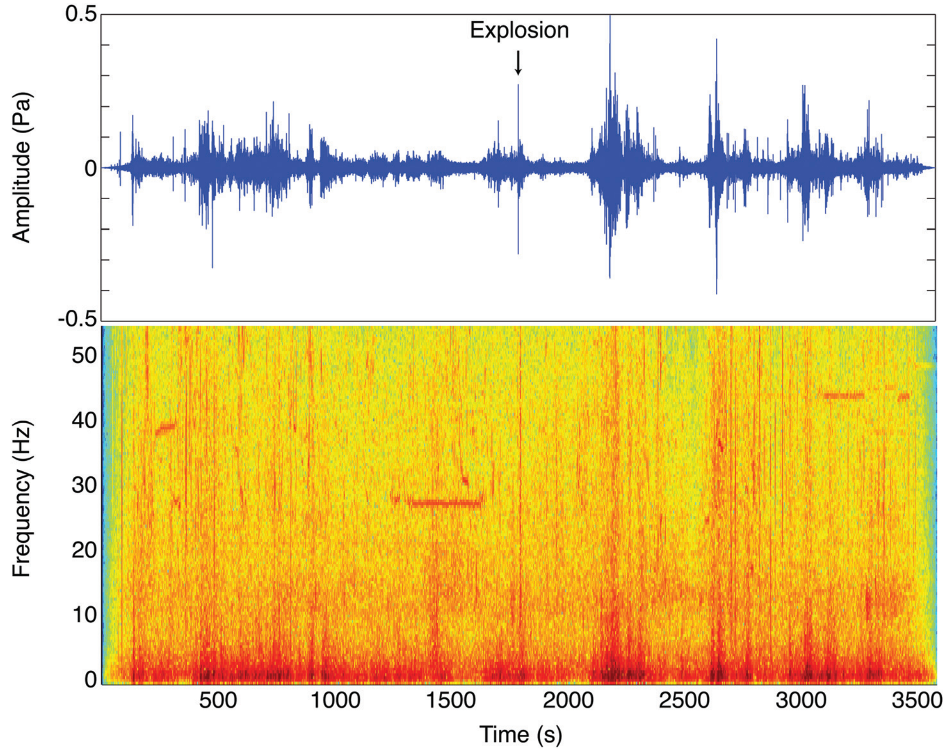

Fig. 1. Explosion signal recorded on a single channel at the recording array and corresponding spectrogram. The explosion signal, which is labeled, cannot be clearly differentiated by eye from the background noise at the scale of the plot in either the time domain or time-frequency domain. The data are high-passed filtered using a Butterworth filter and high-pass cutoff of 1 Hz.

$$X^2 = -2\sum_{i=1}^{k} \ln p_i. \tag{7}$$

Therefore, the sum of the natural logarithm of p-values given by Eq. (7) is given by a χ^2 distribution with $2k$ degrees of freedom where k is the number of discriminants.

4. Dataset

This paper focuses on the acoustic measurement of a 400 g explosion conducted at the Los Alamos National Laboratory Minie explosion test site. We chose to focus on this shot because it represents a signal that is on the margin of detectability for existing separate detectors. During the course of our study, we have examined many shots, which are not discussed in the paper for the sake of conciseness, but which all show an improvement in the detector performance when combining the separate detectors. Of course, for large shots the individual detectors already perform sufficiently and the combination serves only to improve the discriminant power. The recording array (Pajarito Lay Down Yard, PLDY), which was located at a range of 5.1 km from the explosion site, comprised three IFS-3000 series Hyperion infrasound sensors. The Hyperion sensors have a nominal sensitivity of 150 mV/Pa, dynamic range of 120 dB, and flat frequency response over the frequency band studied in this paper (1–100 Hz). Data were digitized by a Reftek RT-130 data acquisition system with a GPS time stamp. The acoustic signal recorded at PLDY was low amplitude (peak pressure ~0.3 Pa) and obscured by environmental and manmade noise associated with traffic, construction, and other sources (Fig. 1), making it a useful case study for this paper. We note that

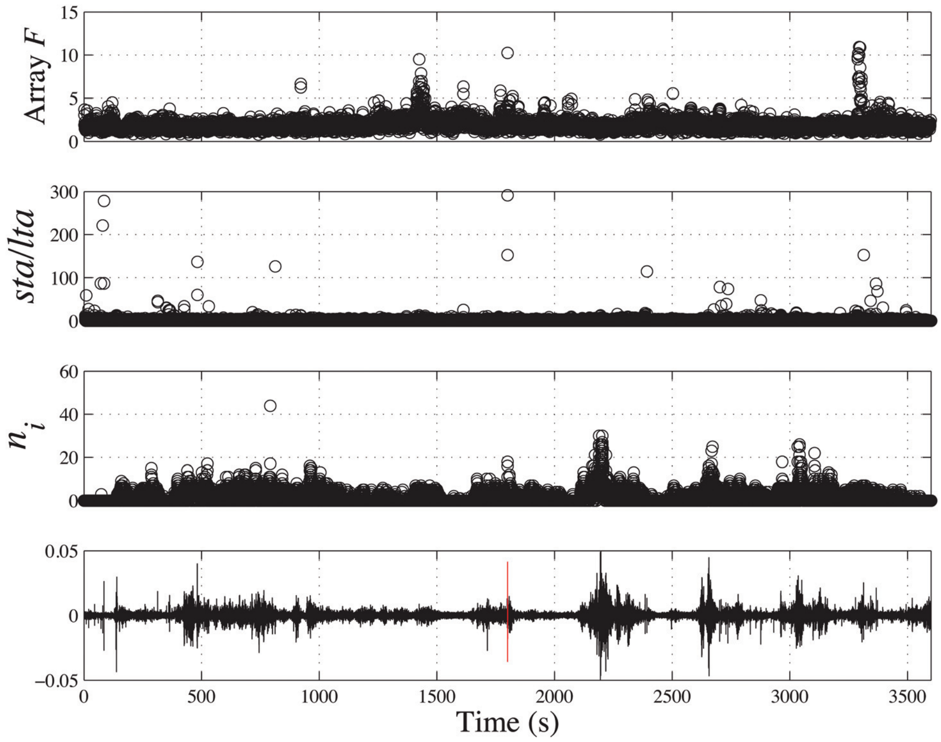

Fig. 2. Detection statistics from top to bottom: Array F, STA/LTA, summed illuminated bins (spectrogram detector) and signal filtered between 1 and 50 Hz (see text for details). Signal arrival shown by the vertical line (red online).

the signal amplitude is lower than that predicted for a free air explosion of the same size recorded at 5.1 km according to standard semi-empirical pressure laws (ANSI, 1983) due to the shot configuration. However, since the focus of this paper is on detection of simple explosions we do not explore configuration effects here. Figure 1 clearly illustrates that the explosion signal is impulsive and broadband, but that there are other impulsive/broadband signals in the record that are not associated with explosions.

5. Results

The individual detection statistics for each of the detectors described in Sec. 3 are shown in Fig. 2. Each detector performs differently and false detections are observed for each in the 1 h time sample. We note that the detectors have been tuned to this signal for the purpose of this paper. Therefore, the results for individual detectors shown here may be better than would be expected in practice, such that the combination of detectors would be essential. Figure 2 shows that all detectors clearly detect the signal, but also detect other events. Individual *p*-values were computed from the detection statistics using Eq. (2) and the combined using Eq. (7). Figure 3 shows the observed X^2 values as a function of time and the horizontal dashed line is the critical value for a detection at $\alpha = 0.99$. The signal is clearly detected and there are no false detections in this 1-h time window.

6. Conclusions

This paper has demonstrated the use of Fisher's Combined Probability test to enhance the detection/discrimination of acoustic signals from small explosions from a complicated background that includes real noise and other types of signal. Each detector/discriminant is quantified in terms of a null hypothesis that is based on a different model of the expected

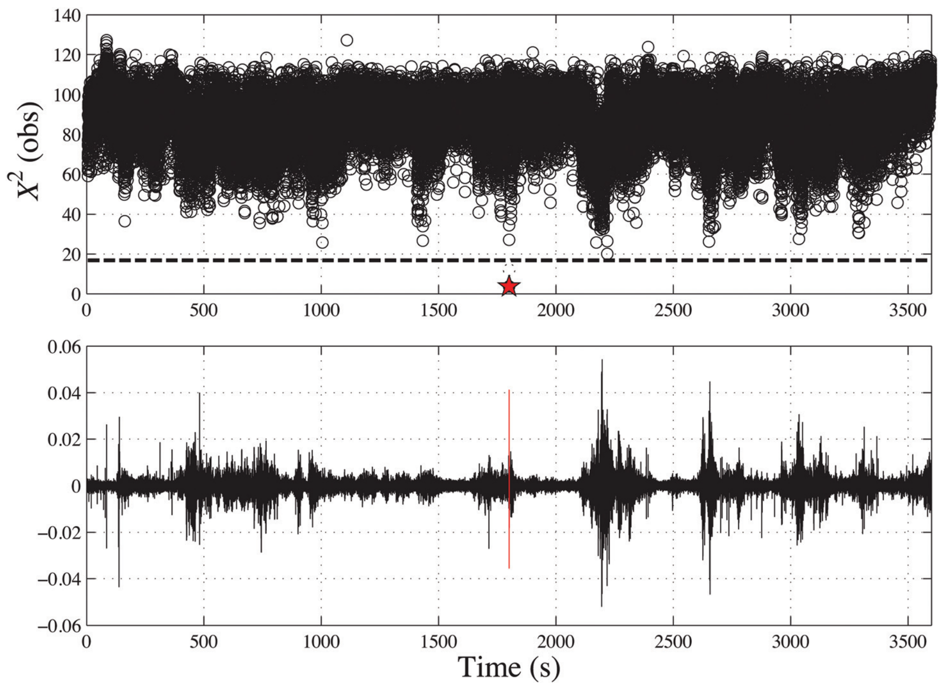

Fig. 3. Results of Fisher's Combined Probability Test. The star indicates signal detection at 0.99 significance level. The horizontal dashed line shows the critical value for a detection at $\alpha = 0.99$.

signal. Because these models are orthogonal (i.e., contain different information on the signal content), combining the detectors using Fisher's Combined Probability test clearly improves the identification of small explosion signals. Such an approach is necessary to adequately detect small explosion signals in complex real-world scenarios.

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